Topic: L'Hospital's rule

Question: Use L'Hospital's rule to evaluate the limit.

$$\lim_{x \to 4} (5 - x)^{\frac{1}{4 - x}}$$

Answer choices:

A 0

B 1

C ∞

D e

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Solution: D

Since we get the indeterminate form 1^{∞} with direct substitution, we let the limit equal y, find the natural log of both sides, and then simplify.

$$y = \lim_{x \to 4} (5 - x)^{\frac{1}{4-x}}$$

$$\ln y = \lim_{x \to 4} \ln \left[(5 - x)^{\frac{1}{4-x}} \right]$$

$$\ln y = \lim_{x \to 4} \frac{1}{4 - x} \ln(5 - x)$$

$$\ln y = \lim_{x \to 4} \frac{\ln(5 - x)}{4 - x}$$

Plugging 4 into the last step above still gives an indeterminate form of $0/0$, so we'll apply L'Hospital's rule.

$$\ln y = \lim_{x \to 4} \frac{\frac{-1}{5 - x}}{-1}$$

Plugging 4 into the last step gives

$$\ln y = 1$$

Raising each side to the base e, we see that the limit is

$$e^{\ln y} = e^1$$

$$y = e$$

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Topic: L'Hospital's rule

Question: Use L'Hospital's rule to evaluate the limit.

$$\lim_{x \to \infty} \frac{e^{2x}}{x^3}$$

Answer choices:

A 0

B 1

C ∞

D $-\infty$

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Solution: C

Since we get the indeterminate form ∞/∞ with direct substitution, we apply L'Hospital's rule until we get a determinate form.

$$\lim_{x\to\infty} \frac{e^{2x}}{x^3}$$

$$\lim_{x\to\infty} \frac{2e^{2x}}{3x^2}$$

still gives an indeterminate form when we evaluate at ∞

$$\lim_{x\to\infty} \frac{4e^{2x}}{6x}$$

still gives an indeterminate form when we evaluate at ∞

$$\lim_{x\to\infty} \frac{8e^{2x}}{6}$$

$\lim_{x\to\infty} \frac{8e^{2x}}{6} = \infty$ when we evaluate at ∞.

Since the last application of the rule allowed us to evaluate the limit by direct substitution without giving us an indeterminate form, we've found that the limit is ∞.

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Topic: L'Hospital's rule

Question: Use L'Hospital's rule to evaluate the limit.

$$\lim_{x \to 1} \frac{\ln x}{x - 1}$$

Answer choices:

A 0

B 1

C ∞

D $-\infty$

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Solution: B

Since we get the indeterminate form $0/0$ with direct substitution, but we can't eliminate the zero in the denominator by factoring, we take the derivative of the numerator and the denominator and then try direct substitution again.

$$\lim_{x \to 1} \frac{\ln x}{x - 1}$$

$$\lim_{x \to 1} \frac{\frac{d}{dx} \ln x}{\frac{d}{dx}(x - 1)}$$

$$\lim_{x \to 1} \frac{1/x}{1}$$

$$\lim_{x \to 1} \frac{1}{x}$$

$$\frac{1}{1}$$

$$1$$

With one application of L'Hospital's rule, we see that the limit is 1.

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